Solar Home Electricity using Natural Convection –

proposal for a mini solar chimney with venturi to multiply air flow velocity and to generate electricity from solar energy with high efficiency

Abstract

Photovoltaics have made possible the massive increase in solar electricity during the last decade with their application to individual homes and community schemes. Convection is one of the vital processes of nature playing a dominant role in the atmosphere, the oceans and in the sun. Is it possible to devise a small solar convection engine that could be used in residential gardens and in rural villages to provide electricity for individual homes and community schemes? Such a model is proposed based on a mini solar chimney and venturi.

A mini solar chimney of height 2 m and cross-sectional area 4 m^2 receives warm air from a solar collector of area 8 m^2. The air flow is required to pass through a venturi where it acquires high velocity. The extra kinetic energy of the air flow comes from the internal energy of the warm air. A turbine sited in the throat of the venturi generates electricity. It is calculated that to achieve an efficiency of 50% the venturi should be of diameter 0.2 m, accelerating the air flow to up to 50 ms^{-1} giving an electrical output of up to 3 kw in bright sunshine. This small model should generate 24 kwh/day in a sunny climate. Larger models are also considered.

The author asks research workers in renewable energy laboratories to build, test and develop such a model. If successful at an economic price, the recommended model could find an enormous market for individual residential gardens in advanced countries. Equally it could provide solar home electricity on a massive scale in individual homes and community schemes in developing countries. Natural convection could provide solar home electricity on the same scale and using parallel consumer mechanisms to those pioneered by photovoltaics.
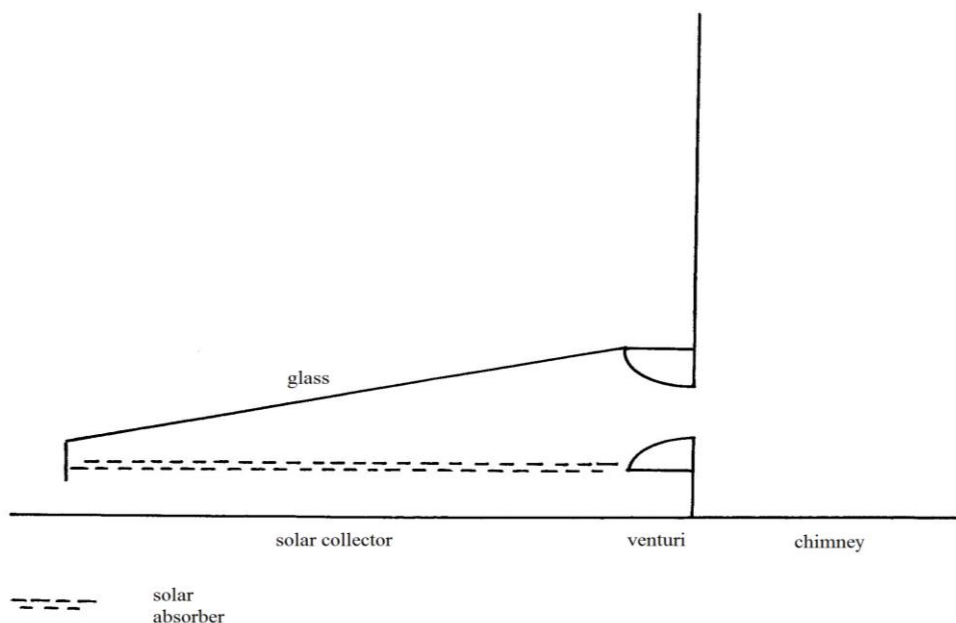
Introduction

Figure 1

Consider Figure 1. The configuration is based on a mini solar chimney of height 2 m and cross-sectional area 4 m^2 and a solar air collector of area 8 m^2. Solar energy passes through the glass and is taken up by the absorber with high efficiency. This warms air in its vicinity which rises because of its lower density. A convection current is established where warm air rises into the chimney and draws replacement air from beneath the absorber. This natural convection flow is required to pass through a venturi or nozzle multiplying flow velocity as it passes through the venturi. On entering the chimney the air flow is drawn through because of its excess temperature.

If a turbine is placed in the throat of the venturi it can harness the kinetic energy of the air flow generating electricity. It is the author's belief that up to 50% of the solar energy absorbed can be converted into flow kinetic energy in the venturi which can be harnessed by a turbine. The other 50% of solar energy absorbed is lost in exit air which leaves the chimney at an elevated temperature. This is a necessary loss as it is the buoyancy of the warm air in the chimney that creates the convection current.

If the above ideas are correct then it should be possible to devise and develop a solar convection engine to generate electricity in the gardens of individual homes for domestic consumption and/or export to the grid as has been pioneered by photovoltaics. Equally such a generator could find widespread application throughout the developing world to provide solar electricity for individual consumption and in small community schemes.

Theoretical Development

Consider the configuration represented in Figure 2. On the left is a rectangular solar air collector whose upper surface is of glass gently sloping upwards towards the chimney. The solar absorber is at a calculated distance above the ground and has two layers with regular large holes to allow easy through flow of air. On the right is a rectangular cross-section well-insulated chimney. This has a roof to prevent back flow, modestly open to allow exit air flow and sloping to use rainwater to wash the glass of the collector. Warm air flow from the collector passes through a nozzle/venturi where it accelerates to high velocity and then flows through a horizontal axis air turbine generating electricity. The warm air post turbine has zero horizontal velocity but is drawn through the chimney by the buoyancy of the air column.
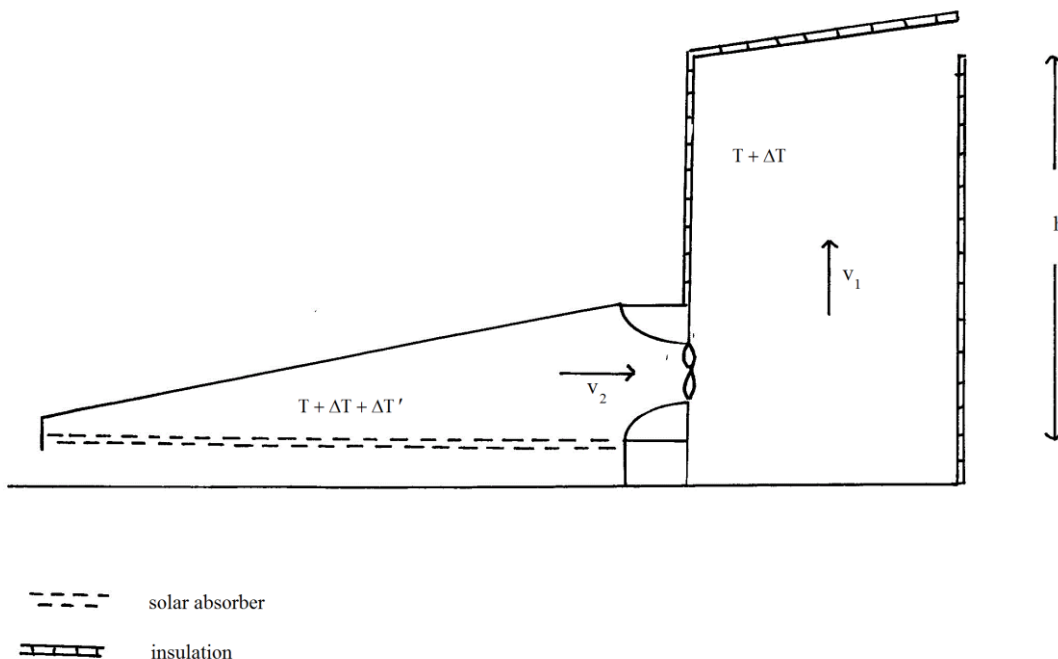

Figure 2

Consider that in Figure 2

h	height from solar absorber to level of exit air
A_1	cross-sectional area chimney
A_2	cross-sectional area turbine
A_3	area solar absorber
v_1	velocity of air flow through chimney
v_2	velocity of air flow through turbine
T	ambient temperature
ΔT	excess temperature (above ambient) exit air
$\Delta T'$	fall in temperature of air flow through turbine
g	gravitational constant
ρ	density of air at atmospheric pressure and temperature T
C_p	heat capacity of air at constant pressure
I	insolation

It is assumed that the turbine is of high efficiency, that exit air leaves the turbine at $T + \Delta T$ and that $\Delta T'$ is used to provide the kinetic energy of air flow through the turbine. The excess temperature of air in the chimney creates buoyancy which draws air through the configuration. The velocity v_1 of air flow through the chimney is given by the solar chimney equation [1, 2]

$$v_1{}^2 = 2 \frac{\Delta T}{T} gh \qquad (1)$$

Constant mass flow requires that

mass flow through chimney = mass flow through the turbine

$$A_1 \, v_1 \, \rho_1 = A_2 \, v_2 \, \rho_2$$

Assuming there is little change in density

$$A_1 \, v_1 = A_2 \, v_2 \qquad (2)$$

As air flow is accelerated through the venturi the gain in kinetic energy comes from the internal energy of the air whose temperature falls by $\Delta T'$

increase in kinetic energy = mass flow x heat capacity x fall in temperature

$$\tfrac{1}{2} \, \dot{m} \, v_2{}^2 = \dot{m} \, C_p \, \Delta T'$$

where $\dot{m}$ is the mass flow. Thus

$$v_2{}^2 = 2 \, C_p \, \Delta T' \qquad (3)$$

The solar energy absorbed provides the mass flow through the collector with an increase in temperature ΔT which creates the buoyancy of the air flow. It also provides a further increase $\Delta T'$ to provide the kinetic energy for air flow through the turbine

solar energy absorbed = mass flow x heat capacity x temperature rise

$$I \, A_3 = \rho \, A_1 \, v_1 \, C_p \, (\Delta T + \Delta T') \qquad (4)$$

Equations (1) – (4) contain 8 variables h A_1 A_2 A_3 v_1 v_2 ΔT $\Delta T'$ and 5 constants T g ρ C_p I. If 4 of the variables are fixed the algebra is soluble. By selecting different values of the variables any number of models can be devised.

Recommended model

Consider that in Figure 2

h	=	2 m		T	=	300 °K
A_1	=	4 m²		g	=	9.81 ms⁻²
A_3	=	8 m²		ρ	=	1.18 kg m⁻³
ΔT	=	ΔT′		C_p	=	1005 j kg⁻¹ K⁻¹
				I	=	750 w m⁻²

A height of 2 m is considered acceptable for residential gardens with a total floor area of 12 m²
for the collector and chimney. It is assumed that $\Delta T = \Delta T'$ so that the model solar engine is
devised for an efficiency of 50%. An ambient temperature 300 °K is considered to replicate warm
climates. The level of insolation assumed is for UK summer maximum.

From equation (1)

$$v_1{}^2 = \frac{2 \, \Delta T \, gh}{T}$$

$$v_1{}^2 = \frac{2 \, \Delta T \times 9.81 \times 2}{300}$$

$$v_1{}^2 = 0.1308 \, \Delta T$$

From equation (4)

$$I A_3 = \rho A_1 v_1 C_p (\Delta T + \Delta T')$$

$$750 \times 8 = 1.18 \times 4 \times 1005 \, v_1 (\Delta T + \Delta T)$$

$$v_1 \Delta T = \frac{750 \times 8}{1.18 \times 4 \times 1005 \times 2}$$

$$= 0.6324$$

Thus

$$v_1{}^3 = 0.1308 \times 0.6324$$

$$= 0.08272$$

$$v_1 = 0.4357 \text{ ms}^{-1}$$

$$\Delta T = 1.451 \text{ °K}$$

From equation (3)

$$v_2{}^2 = 2 C_p \Delta T'$$

$$= 2 \times 1005 \times 1.451$$

$$v_2 = 54.01 \text{ ms}^{-1}$$

From equation (2)

$$A_1 v_1 = A_2 v_2$$

$$A_2 = \frac{4 \times 0.4357}{54.01}$$

$$= \quad 0.03227 \text{ m}^2$$

This gives a turbine diameter of 0.2027 m

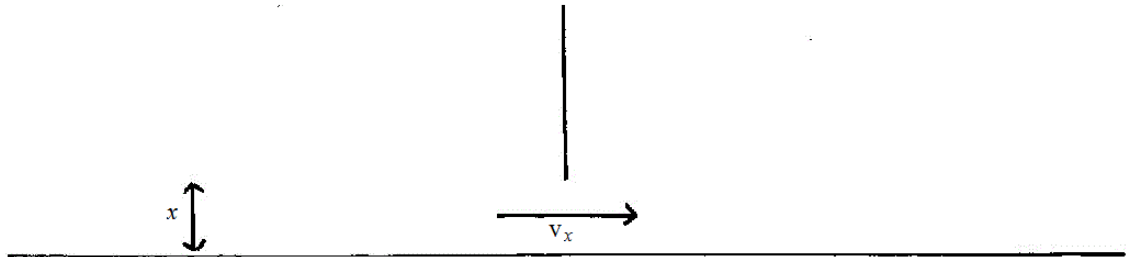**Clearance beneath solar collector**

Figure 3

Consider that the entrance for incoming air into the solar collector is of width w and height x above the ground and that incoming air has a velocity v_x

rate of flow of incoming air $=$ rate of flow through turbine

$$w \; x \; v_x = \quad v_2 \, A_2$$

In the above calculation $w = 2 \quad A_2 = 0.03227$

It is suggested that it would be reasonable to have $v_x = 0.1 \, v_2$

$$x \quad = \quad \frac{0.03227 \, v_2}{2 \; x \; 0.1 \, v_2}$$

$$x \quad = \quad 0.16 \text{ m}$$

Thus there should be a minimum air gap of 0.16 m between the solar collector and the ground.

CHECK

maximum insolation	$I \, A_3$	$=$	750 x 8
		$=$	6000 watts
maximum kinetic energy through turbine		$=$	$\frac{1}{2} \, \rho \, A_2 \, v_2^{\,3}$
		$=$	$\frac{1}{2}$ x 1.18 x 0.03227 x $(54.01)^3$
		$=$	3000 watts
heat loss in exit air		$=$	$\rho \, A_1 \, v_1 \, C_p \, \Delta T$
		$=$	1.18 x 4 x 0.4357 x 1005 x 1.451
		$=$	2999 watts

Thus the model suggested provides maximum kinetic energy of 3 kw. If built in a sunny climate with insolation 6 kwh/m^2/day it has a daily output of 24 kwh. Allowing for the lower insolation in the UK the output is about equivalent to average UK residential demand. In the developing world the model described could be the basis of community schemes each meeting the electricity demand of 10-20 homes. The model is drawn to scale in Figure 4.
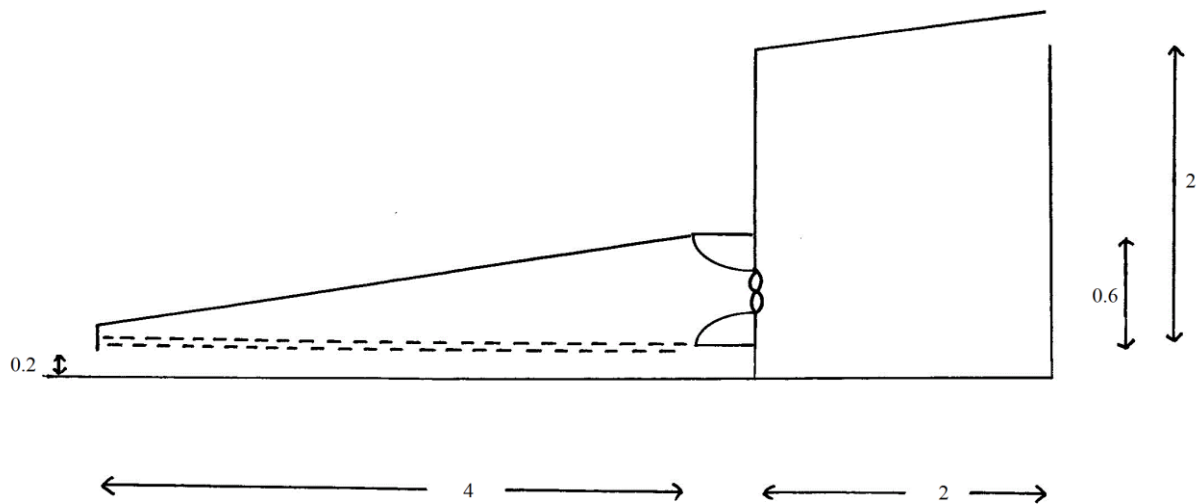

Figure 4

Economics of Recommended Model

Consider that the solar electricity generated is of value $0.10 kwh. Then daily generation of 24 kwh is of value $2.40 or $876 year.

The author's guesstimates of cost are

3 kilowatt turbine	3000
8 m^2 solar collector	800
chimney, nozzle, electrical ..	1200
total cost	$ 5000

This would give a repayment period of 6 years.

Varying solar collector area

The values selected for A_1 and A_3 in the recommended model were chosen to give convenient construction and output. Smaller or larger models could be devised. Table 1 has been drawn up with $h = 2$ m, $A_1 = 4$ m^2 and $\Delta T = \Delta T'$ in each case.

A_3	4	6	8	10	20	m^2
v_1	0.3458	0.3959	0.4357	0.4694	0.5914	ms^{-1}
v_2	42.87	49.07	54.01	58.18	73.31	ms^{-1}
ΔT	0.9144	1.198	1.451	1.684	2.674	°K
output max	1.5	2.25	3.0	3.75	7.5	kw
output daily	12	18	24	30	60	kwh

Table 1

Calculation gives $A_2 = 0.03227$ m^2 giving turbine diameter 0.2027 m for each model. Also the clearance x (the gap between the collector and the ground) is a minimum 0.16 m in each case giving overall height 2.2 m.

All the above configurations have been calculated for efficiency 50% with constant chimney dimensions 2 x 2 x 2 m. As the area of the solar collector is increased from 4 to 20 m^2, the maximum velocity of air flow through the turbine is increased from 40 to 70 ms^{-1}. The excess temperature of exit air increases from 0.9 to 2.7 °K. For the largest collector in a sunny climate there is a daily output of 60 kwh.

Larger model systems

In Table 1 a height $h = 2$ m was chosen as an acceptable height for solar home electricity systems in residential areas. But larger models could be developed as illustrated in Table 2.

Table 2 has been drawn up with $\Delta T = \Delta T'$ to give 50% efficiency, with $A_1 = h^2$ and with $A_3 = 2h^2$ giving reasonable geometric dimensions. If the solar collector is of larger area than this relationship v_2, ΔT and output are increased. Conversely if the cross-sectional area of the chimney A_1 is made relatively larger it will give lower v_2 and ΔT.

h	1	2	3	5	10	m
A_1	1	4	9	25	100	m^2
A_3	2	8	18	50	200	m^2
v_1	0.3458	0.4357	0.4988	0.5914	0.7451	ms^{-1}
v_2	60.63	54.01	50.48	46.36	41.31	ms^{-1}
ΔT	1.829	1.451	1.268	1.069	0.8488	$^\circ K$
A_2	0.005704	0.03227	0.08892	0.3189	1.804	m^2
turbine diameter	0.08524	0.2027	0.3366	0.6374	1.516	m
output max	0.75	3.00	6.75	18.75	75	kw
output daily	6	24	54	150	600	kwh
x	0.06	0.16	0.30	0.64	1.80	m
overall height	1.1	2.2	3.3	5.7	12	m

Table 2

The model using h = 1 could be developed as a laboratory demonstration. It has high v_2 and ΔT, an output of 6 kwh/day and an overall height of 1.1 m.

The model using h = 2 is strongly recommended by the author as optimal for development with a convenient overall height of 2.2 m suitable for residential areas.

The model using h = 3 has overall height 3.3 m and daily output 54 kwh. This would be sufficient for 2 or more homes in advanced countries or 20-30 homes in developing countries. It could be well suited for modest rural village schemes.

The model using h = 5 has overall height 5.7 m and daily output 150 kwh. This could again be well suited for community schemes.

When we consider a model with height 10 m the clearance under the collector increases disproportionately to 1.8 m giving an overall height of 12 m. The daily output however rises to 600 kwh.

As we increase the height of the chimney to 10 m and above, the clearance needed under the collector becomes disproportionately larger. The turbine diameter also increases rapidly. The latter could be offset by using 3 or 4 smaller turbines arranged symmetrically at the same level.

Tables 1 and 2 demonstrate that there is considerable potential for devising larger models with higher output. It is the author's hope however that the recommended model should first be built, tested and developed leading to a wide array of larger models for future development.

Alternative Configuration

A radically different alternative is possible from the model described hitherto. In Figure 5 the nozzle and turbine are built into the lower half of a cylindrical chimney. The turbine has a vertical axle. Calculations are identical to those outlined above and the results in Tables 1 and 2 also apply.
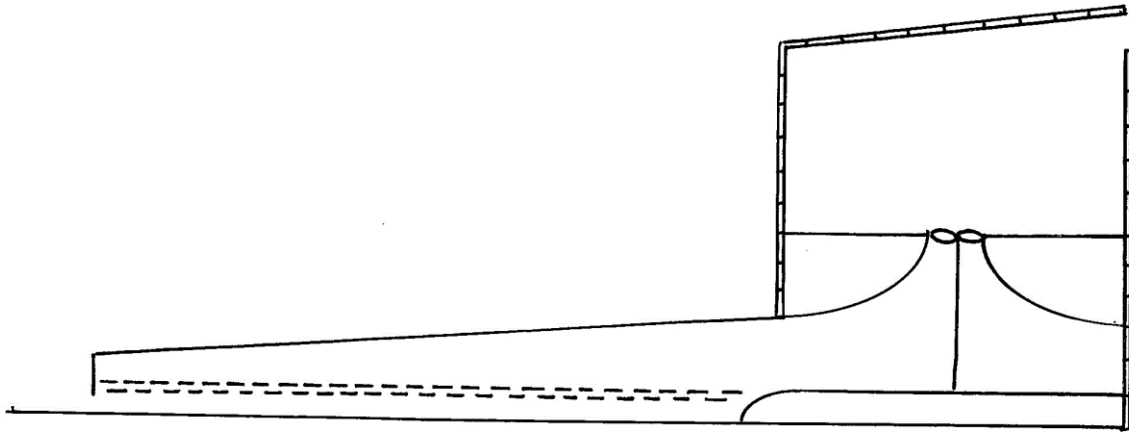
Figure 5

Possible advantages of this alternative configuration:

- airflow from the solar collector leads directly to the chimney allowing a lower height and gentler slope for the upper surface of the collector.

- the entire cross-sectional area of the chimney is available for the mouth of the nozzle allowing smoother acceleration of the air flow pre-turbine.

- the solar collector could be circular around the chimney.

Advantages of the earlier configuration:

- there is greater experience in the construction, operation and maintenance of horizontal axis wind turbines and greater availability.

- the nozzle and turbine are near ground level.

Additional Comments

- Energy losses – no allowance has been made in the calculations for energy losses through the solar collector, turbine, generator … The author has devised parameters such that ΔT and $\Delta T'$ are below 2 ºC to minimise losses in the collector. Any energy losses in the turbine will be manifested as heat in the chimney and will contribute to buoyancy – thus turbine/generator losses are effectively recycled. If there are significant energy losses, the area of the solar collector can be increased to raise output.

- Energy storage – sealed water tubes with a depth of water of up to 20 cm can be used for storage of daytime solar energy to allow evening and night generation [2, 3]. For larger versions of this proposal of height 3 m and above such water tubes could be arranged at ground level beneath the solar absorber. The value of x – the clearance beneath the absorber in Table 2 shows increasingly abundant available volume for energy storage in larger models.

- Turbine – conventional horizontal axis wind turbines are designed for wind velocities of up to 15 ms^{-1}. The air flow velocities involved in these proposals are considerably larger at 40-70 ms^{-1} implying 20-100 times the power density. This will require specially designed turbines.

- Nozzle – the nozzle/venturi in the recommended model has throat diameter 0.2 m and mouth diameter about 0.6 m. It accelerates air flow to 50 ms^{-1} over a distance of under 1 m. It needs to be built of solid strong material and be fixed rigidly in place.

Conclusion

A mini solar chimney of height 2 m and cross-sectional area 4 m^2 receives warm air from a solar collector of area 8 m^2. The air flow, generated by natural convection, is required to pass through a venturi/nozzle to multiply flow velocity. A horizontal axis turbine sited in the throat of the venturi can harness the kinetic energy of the air flow. A model has been devised for the conversion of solar energy into electricity with an efficiency of 50%. It has turbine diameter 0.2 m, maximum air flow velocity 50 ms^{-1} and maximum output 3 kw.

The author asks research workers in renewable energy laboratories to build, test and develop the model proposed. It could provide solar electricity using natural convection at an economic price in the gardens of residential properties in advanced countries. Equally it could find widespread application in individual homes and in community electricity schemes in developing countries.

References

[1] Mullett, L. B. The solar chimney – overall efficiency, design and performance. Int. J. Ambient Energy 1987, 8, 35-40.

[2] Schlaich, J., Bergermann, R., Schiel, W. and Weinrebe, G. Design of Commercial Solar Updraft Power Systems – Utilization of Solar Induced Convective Flows for Power Generation. J. Sol. Energy Eng. 2005, 127, 117-124.

[3] Xinping Zhou, Yangyang Xu. Solar updraft tower power generation. Solar Energy, 128, (2016) 95-125.

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