Divergent Solar Chimney – dimensions for a possible prototype

Abstract

Several recent publications have come to the conclusion that divergent solar chimneys have an efficiency many times that of their cylindrical counterparts. These are briefly reviewed and the importance of area ratio (area top of chimney divided by area base of chimney) and divergent angle highlighted. A theoretical model is presented and dimensions considered for a prototype DSC. This has height 50 m, solar absorber area 500 m^2, top chimney diameter 9 m and base chimney diameter 2 m. It is calculated that the prototype DSC will generate 150 kw at maximum insolation with an efficiency of 40%. Divergent solar chimneys of up to 300 m height and larger capacity are also considered.

Introduction

Of the solar energy that is intercepted by the earth, one half is reflected into space and one half is absorbed by the surface layers of the ground and the oceans. This warms air at the base of the atmosphere; the warm air is lighter and rises due to gravitation. This continuous current of rising air and the rotation of the earth create wind and weather. It is a paradox that convection is the dominating process for harnessing solar energy by the earth but plays virtually no part in the global effort to generate electricity from solar energy.

The solar chimney [1] is the only example of the use of natural convection to drive turbines to generate electricity. Efficiency is proportional to the height of the chimney but even with a 1000 m height chimney, efficiency is only 2-3%. There have been several studies into whether the shape of the chimney e.g. cylindrical, convergent or divergent, might influence efficiency but, over the decades, results have been inconclusive. In recent years, however, there has been growing evidence that divergent solar chimneys have enhanced efficiency.

Koonsrisuk and Chitsomboon [2] used CFD technology to investigate the effect of a sloping collector and a divergent-top chimney. They found that with a height of 200 m and area ratio of 16 this can produce power as much as 400 times that of the cylindrical chimney. There has been no detailed follow-up to this paper but its conclusions are quite startling.

Ohya et al. [3] carried out experimental work on a 400 mm height solar chimney and found that the diffuser type tower with a 4 deg open ratio induced an updraft velocity of 1.38 – 1.44 times more than the cylindrical type tower. This indicates a 2.6 – 3.0 increase in power output.

The present author [4] developed a theoretical model for a divergent solar chimney and found that the efficiency is multiplied by $(A_1/A_2)^2$ when compared to the cylindrical SC where A_1 is the cross-sectional area of the top of the chimney and A_2 the area of the base of the chimney. A prototype DSC of height 100 m was proposed but this must now be withdrawn as it took no account of the 4° divergence angle limit suggested by Ohya [3]. Subsequent communications [5] have highlighted problems of flow separation/diffuser stall above an optimum 4° angle.

Pattanashetti and Madhukeshwara [6] write that it appears that the kinetic energy at the base of the DSC might increase in proportion to the square of the tower area ratio. Using CFD analysis they found a higher flow rate and a lower temperature rise in the DSC and conclude that a tower area ratio of 12 to 16 can produce kinetic energy as much as 100 times that of the constant area tower.

Ohya et al. [7] tested various diffuser shapes by laboratory experiments and numerical analyses and found that a diffuser tower with a semi-open angle of 4° is an optimal shape producing the fastest updraft. Using a 2 m high tower and 4° angle the updraft speed was greater than for a cylindrical tower by a factor of 1.5 to 1.7. By inserting a wind turbine, power outputs obtained in the diffuser tower were 4 – 5 times greater than that of the cylindrical tower. Ohya et al. are now conducting experimental work on a prototype solar tower of 10 m height.

Leung et al. [8] used CFD to investigate the divergent solar chimney and found that the power multiple of divergent versus cylindrical increases with the area ratio (AR) but falls away at high AR. This parabolic tendency is associated with diffuser stall. Using the Manzanares 200 m high SC as benchmark they found that power output could be multiplied by about 13.5 with an area ratio of 10.

Leung et al. [9, 10] used CFD analysis to compare divergent solar chimneys of height 100 – 300 m and area ratio 0 to 32. Again they found that very high AR give problems of diffuser stall/ flow separation but with an area ratio of 9 – 12 normalized power output is multiplied by 10 – 24 divergent compared to cylindrical solar chimney.

Zhou et al. [11] used a theoretical model to consider the effect of flow area on available power in a solar chimney. Divergent-top shape is recommended and an upward slanting roof shape of the solar collector. The mass flow rate increases with the reduction of the collector inlet area. The increase is seen to be clear with the reduction of the chimney inlet area and clearer with the increase of the chimney outlet area.

Against this accumulating evidence of substantial advantage from the use of the divergent chimney and using his own theoretical model, the present author has attempted to devise dimensions for a prototype divergent solar chimney and to estimate its possible performance.

Theoretical Development

Consider Figure 1 which represents a divergent solar chimney

h	height of chimney
A_1	area cross-section top of chimney
A_2	area cross-section base of chimney
A_3	area solar absorber
v_1	velocity of air flow at top of chimney
v_2	velocity of air flow through turbine
T	ambient temperature
ΔT	excess temperature (above ambient) of exit air
$\Delta T'$	fall in temperature as air flows through turbine
g	gravitational constant
ρ	density of air at atmospheric pressure and temperature T
C_p	heat capacity of air at constant pressure and temperature T
I	insolation

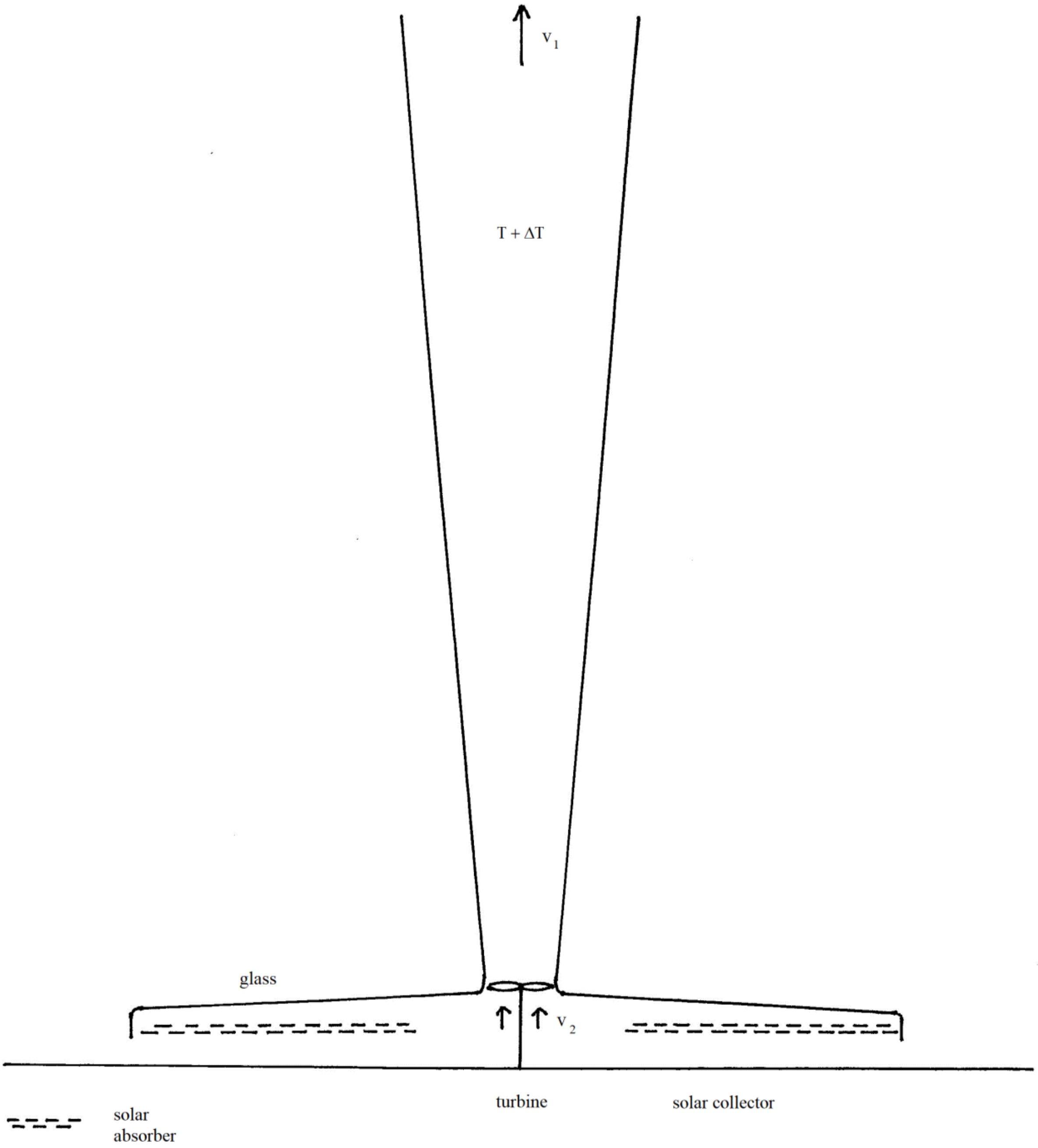

Figure 1

The absorber takes up solar energy with high efficiency warming air in its vicinity which rises. The flow of warm air accelerates to pass through the constricted area at the base of the chimney. The turbine is sited at the level of maximum constriction to harness the kinetic energy of the air flow. The warm air leaves the chimney at a temperature $T + \Delta T$ and with velocity v_1 given by the solar chimney equation (1)

$$v_1^{\,2} = 2 \frac{\Delta T}{T} gh \qquad (1)$$

Assuming little change in density, constant mass flow requires that

$$A_1 v_1 = A_2 v_2 \qquad (2)$$

As air accelerates into the turbine it gains kinetic energy but suffers a fall in temperature

gain in kinetic energy = mass flow x heat capacity x fall in temperature

$$\tfrac{1}{2}\,\dot{m}\,v_2^{\,2} = \dot{m}\,C_p\,\Delta T'$$

where ṁ is the mass flow

$$v_2^{\,2} = 2\,C_p\,\Delta T' \qquad (3)$$

As air flows through the solar absorber, its temperature rises by an amount ΔT which is lost in exit air and by an amount ΔT′ which provides kinetic energy to the turbine.

total solar energy absorbed = mass flow x heat capacity x temperature rise

$$I\,A_3 = \rho\,A_1\,v_1\,C_p\,(\Delta T + \Delta T') \qquad (4)$$

These 4 equations define the physics of the divergent solar chimney. They contain 5 constants T g ρ C_p I and 8 unknowns h A_1 A_2 A_3 v_1 v_2 ΔT $\Delta T'$. If 4 of the unknowns are fixed the algebra is soluble.

Design of a possible prototype divergent solar chimney

Consider a chimney of height 50 m, divergent angle 4°, base chimney radius r and top of chimney radius R as shown in Figure 2

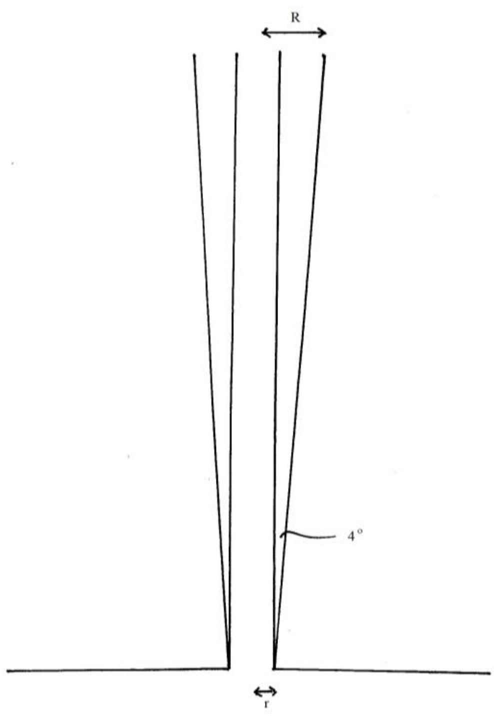

Figure 2

Simple trigonometry gives

$$\begin{aligned} R &= r + 50 \tan 4 \\ &= r + 50 \times 0.06993 \\ &= r + 3.496 \end{aligned}$$

Various values of r have been considered and it is suggested that the prototype DSC have base diameter 2 m, top diameter 9 m giving a divergent angle of 4.004°. Thus in equations (1) – (4) consider

h	=	50 m	T	=	300 °K
A_1	=	63.59 m^2 (diameter 9 m)	g	=	9.81 ms^{-2}
A_2	=	3.14 m^2 (diameter 2 m)	ρ	=	1.18 kg m^{-3}
A_3	=	500 m^2	C_p	=	1005 j kg^{-1} K^{-1}
			I	=	750 w m^{-2}

Consider solar absorber area 500 m^2 giving a broadly proportionate configuration, ambient temperature 300 °K and insolation 750 w m^{-2} representing UK summer maximum.

From equation (1)

$$v_1^{\,2} = \frac{2\ \Delta T}{300} \times 9.81 \times 50$$

$$v_1^{\,2} = 3.27\ \Delta T \qquad (1)$$

From equation (2)

$$63.59\ v_1 = 3.14\ v_2$$

$$v_2 = 20.25\ v_1 \qquad (2)$$

From equation (3)

$$v_2^{\,2} = 2 \times 1005\ \Delta T'$$

$$v_2^{\,2} = 2010\ \Delta T' \qquad (3)$$

From equation (4)

$$750 \times 500 = 1.18 \times 63.59 \times 1005\ v_1 (\Delta T + \Delta T')$$

$$v_1 (\Delta T + \Delta T') = 4.973$$

From (1) (2) and (3) above

$$v_1 \left(\frac{v_1^{\,2}}{3.27} + \frac{410.1}{2010}\ v_1^{\,2} \right) = 4.973$$

$$v_1^{\,3}\ (2010 + 1341) = 4.973 \times 3.27 \times 2010$$

$$v_1^{\,3} = 9.755$$

$$v_1 = 2.137$$

From (1) $\Delta T = 1.396$

From (2) $v_2 = 43.27$

From (3) $\Delta T' = 0.9314$

The above values are for insolation 750 w m^{-2} representing total solar energy absorbed of 500 x 750 = 375 kw. The amount of kinetic energy provided for the turbine is

$$\frac{1}{2} \rho A_2 v_2^{\,3} = \frac{1.18}{2} \times 3.14 \times (43.27)^3$$

$$= 150.1 \text{ kw}$$

This represents an efficiency of

$$\frac{150.1}{375} = 40.02\%$$

It is quite astounding that an efficiency of 40% may be achievable in the conversion of solar energy into electricity using a solar chimney and of only 50 m height. But it is a divergent solar chimney with A_1/A_2 20.25 and whose efficiency is thus multiplied by $(20.25)^2$ = 410.1 compared to the cylindrical solar chimney. The dimensions considered conform to the divergent angle 4° and the maximum output of 150 kw could be economically viable.

The author has considered scaling up output by using a larger area solar absorber. This is possible but it would mean higher ΔT, $\Delta T'$ and v_2.

<u>Clearance under the collector</u>

Consider that the bottom edge of the solar collector is at a height *x* above the ground, that the diameter of the solar collector is D and that incoming air has a velocity v_x (Figure 3)

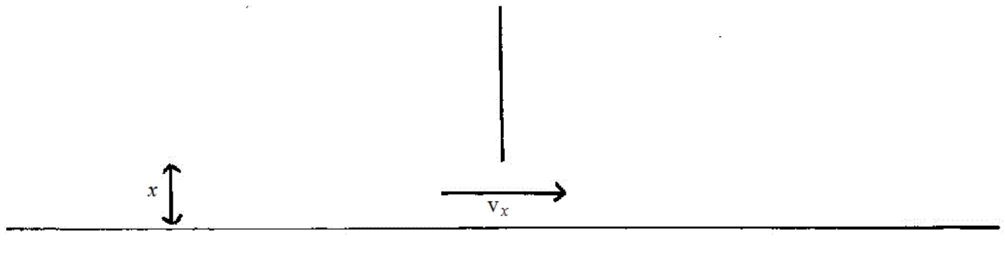

Figure 3

rate of flow of incoming air = rate of flow through turbine

$$\pi D x \; v_x = v_2 A_2$$

In the prototype above

area solar absorber = 500 m^2

area chimney = 63.59 m^2

area solar collector = 563.6 m^2

diameter solar collector = 26.79 m

An incoming air velocity $v_x = 0.1\ v_2$ is suggested giving a maximum velocity of about 4 ms^{-1}

$$x = \frac{3.14\ v_2}{3.14 \times 26.79 \times 0.1\ v_2}$$

$$= 0.3732\ \text{m}$$

Thus a clearance of about 0.4 m is needed between the bottom edge of the solar collector and the ground.

Design of Larger Divergent Solar Chimneys

Calculations have been carried out for a variety of possible DSC of height 50 – 300 m following exactly the same steps as above. In each case the divergent angle is 4.004°. The solar absorber area $A_3 = 0.2\ h^2$ has been selected to give proportionate size. The results are presented in Table 1

h	50	100	150	200	250	300
top diameter	9	19	29	40	52	64
base diameter	2	5	8	12	17	22
A_1	63.59	283.4	660.2	1256	2123	3215
A_2	3.14	19.63	50.24	113.0	226.9	379.9
A_1/A_2	20.25	14.44	13.14	11.11	9.356	8.463
A_3	500	2000	4500	8000	12,500	18,000
v_1	2.137	2.591	2.842	3.080	3.289	3.445
v_2	43.27	37.41	37.34	34.22	30.77	29.15
ΔT	1.396	1.026	0.8232	0.7252	0.6615	0.6049
$\Delta T'$	0.9314	0.6964	0.6938	0.5826	0.4710	0.4229
efficiency %	40.02	40.42	45.73	44.55	41.59	41.14
maximum output kw	150.1	606.3	1543	2673	3899	5555
x	0.3732	1.159	1.973	3.315	5.294	7.360

Table 1

The dimensions for each configuration have been devised with a target efficiency of 40-45%. Higher efficiency is possible by reducing the base and top diameter in line with the 4° constraint but higher efficiency means increasing ΔT, $\Delta T'$ and v_2. Output can also be increased in any configuration by increasing A_3 but this also implies increasing ΔT, $\Delta T'$ and v_2.

Additional Comments

- The divergent solar chimney as represented in Figure 1 is a highly unstable structure. It could be built with an exterior cylindrical framework to provide strength and stability. Its appearance would then be the same as that of a cylindrical solar chimney.

- The junction between the solar collector and the base of the chimney has been presented as an abrupt, slightly obtuse angle in Figure 1. In practice this needs to be smoother with a curved, nozzle-like design leading from the collector to the chimney.

- No allowance has been made for energy losses through the glass of the solar collector or in the turbine. The temperature of warm air in the collector is only 1 - 2°C above ambient minimizing heat losses. Any energy losses in the turbine will be manifested as heat and will contribute to the buoyancy of air flow through the chimney. They are effectively recycled.

- Energy storage can easily be added using water tubes at ground level underneath the absorber. [1]

- The divergence angle limit of 4° imposes a severe constraint on the A_1/A_2 ratio at a given height. This indirectly imposes a very small base of chimney diameter which severely limits the output capacity of the divergent solar chimney. There may be methods of tackling flow separation e.g. swirl, use of guide vanes … that allow an angle of 5 - 6 - 8° … and thus much larger solar collector area, turbine size and output.

Conclusion

Dimensions for a possible prototype divergent solar chimney are outlined. It is of height 50 m, solar absorber area 500 m^2, top chimney diameter 9 m and base chimney diameter 2 m. Calculations indicate an output of 150 kw at maximum insolation with an efficiency of 40%. The author asks experts on the solar chimney to conduct CFD and other theoretical analysis to assess the proposal.

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